ATIVIDADE:

Resolução comentada - LISTA 11

01.

$$(x+p)^3 = \binom{3}{0}x^3 - 4\binom{3}{1}x^2 + 16\binom{3}{2}x - 64\binom{3}{3}$$

$$(x+p)^3 = \binom{3}{0}x^3 \cdot 1 + \binom{3}{1}x^2 \cdot (-4) + \binom{3}{2}x^1 \cdot 16 + \binom{3}{3}x^0 \cdot (-64)$$

$$(x+p)^3 = \binom{3}{0}x^3 \cdot (-4)^0 + \binom{3}{1}x^2 \cdot (-4)^1 + \binom{3}{2}x^1 \cdot (-4)^2 + \binom{3}{3}x^0 \cdot (-4)^3$$

$$(x+p)^3 = (x+(-4))^3$$

Comparando, temos que $p = -4$. Gabarito A.

02. Gabarito C.

$$\begin{aligned} E &= (999)^5 + 5\cdot(999)^4 + 10\cdot(999)^3 + 10\cdot(999)^2 + 5\cdot(999) + 1 \\ &= 1\cdot(999)^5\cdot 1^0 + 5\cdot(999)^4\cdot 1^1 + 10\cdot(999)^3\cdot 1^2 + 10\cdot(999)^2\cdot 1^3 + 5\cdot(999)\cdot 1^4 + 1\cdot(999)^0\cdot 1^5 \\ &= \binom{5}{0}(999)^5\cdot 1^0 + \binom{5}{1}(999)^4\cdot 1^1 + \binom{5}{2}(999)^3\cdot 1^2 + \binom{5}{3}(999)^2\cdot 1^3 + \binom{5}{4}(999)^1\cdot 1^4 + \binom{5}{5}(999)^0\cdot 1^5 \\ &= (999+1)^5 \\ &= 1000^5 \\ &= (10^3)^5 \\ &= 10^{15} \end{aligned}$$

03. Gabarito E.

$$\begin{aligned} E &= 153^4 - 4\cdot 153^3\cdot 3 + 6\cdot 153^2\cdot 3^2 - 4\cdot 153\cdot 3^3 + 3^4 \\ &= 1\cdot 153^4\cdot(-3)^0 + 4\cdot 153^3\cdot(-3)^1 + 6\cdot 153^2\cdot(-3)^2 + 4\cdot 153^1\cdot(-3)^3 + 1\cdot 153^0\cdot(-3)^4 \\ &= \binom{4}{0}153^4\cdot(-3)^0 + \binom{4}{1}153^3\cdot(-3)^1 + \binom{4}{2}153^2\cdot(-3)^2 + \binom{4}{3}153^1\cdot(-3)^3 + \binom{4}{4}153^0\cdot(-3)^4 \\ &= (153+(-3))^4 \\ &= 150^4 \\ &= (15\cdot 10)^4 \\ &= 15^4\cdot 10^4. \end{aligned}$$

04. Note que

$$x^4+4x^3y+6x^2y^2+4xy^3+y^4=16$$

$$\binom{4}{0}x^4\cdot y^0+\binom{4}{1}x^3\cdot y^1+\binom{4}{2}x^2\cdot y^2+\binom{4}{3}x^1\cdot y^3+\binom{4}{4}x^0\cdot y^4=16$$

$$(x+y)^4=16$$

Logo, $x+y=\pm\sqrt[4]{16}\Rightarrow x+y=\pm 2$. Como x e y são números positivos, então $x+y=2$. Do sistema $\begin{cases} x+y=2 \\ x-y=1 \end{cases}$, segue que $x=\dfrac{3}{2}$. Gabarito E.

05. Gabarito D.

$$\begin{aligned}
\text{E} &= 103^4-4\cdot 103^3\cdot 3+6\cdot 103^2\cdot 3^2-4\cdot 103\cdot 3^3+3^4\\
&=1\cdot 103^4\cdot(-3)^0+4\cdot 103^3\cdot(-3)^1+6\cdot 103^2\cdot(-3)^2+4\cdot 103^1\cdot(-3)^3+1\cdot 103^0\cdot(-3)^4\\
&=\binom{4}{0}\cdot 103^4\cdot(-3)^0+\binom{4}{1}\cdot 103^3\cdot(-3)^1+\binom{4}{2}\cdot 103^2\cdot(-3)^2+\binom{4}{3}\cdot 103^1\cdot(-3)^3+\binom{4}{4}\cdot 103^0\cdot(-3)^4\\
&=(103+(-3))^4\\
&=100^4\\
&=\left(10^2\right)^4\\
&=10^8
\end{aligned}$$